Mechanical Vibration

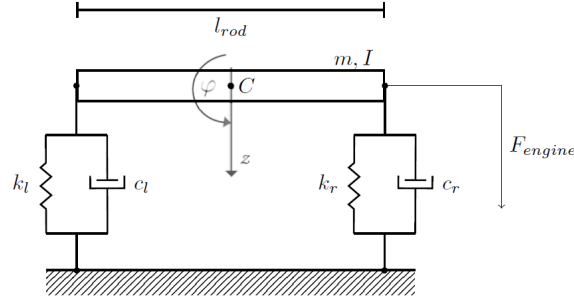## Multi Degree of Freedom DAMPED SYSTEM

Damped Symmetrical Vehicle Suspension with 2DOF for $c_r = k_r\lambda$, $c_l = k_l\lambda$, $l_l = l_r$, $k_l = k_r$

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Scheme of the system

Scheme of the real object is presented in the figure. It was obtained by the analysis of real mechanism.

Analysis of the scheme allows to claim its number of degrees of freedom which is 2

Table with parameter values for calculations

The values of individual parameters adopted for the calculations are presented in the table 1

Tabela 1: Default parameter values

Parameter	Value
c_r	$k_r \lambda$
c_l	$k_l \lambda$
l_l	l_r
k_l	k_r

Kinetic energy

Kinetic energy of the system has a following form:

$$T = \frac{I \dot{\varphi}^2}{2} + \frac{m \dot{z}^2}{2} \tag{1}$$

Determined formula specify energy of the system related to its inertial properties.

Potential energy

Potential energy of the system has a following form:

$$V = \frac{k_r \left(l_r \varphi - z\right)^2}{2} + \frac{k_r \left(-l_r \varphi - z\right)^2}{2} \tag{2}$$

The presented relationship describes the interaction of potential force fields in which the object is located.

Dissipative Rayleigh function

The energy dissipated by attenuation is given by the formula:

$$D = \frac{k_r \lambda \left(l_r \dot{\varphi} + \dot{z}\right)^2}{2} + \frac{k_r \lambda \left(-l_r \dot{\varphi} + \dot{z}\right)^2}{2} \tag{3}$$

The given dependence is the Rayleigh dissipation potential, which when differentiated against the generalized velocity vector allows to determine the viscous damping forces.

Lagrangian of the system (Lagranges function)

System Lagrangian is described by the formula (4):

$$L = \frac{I\dot{\varphi}^2}{2} + \frac{m\dot{z}^2}{2} - k_r z^2 - k_r l_r^2 \varphi^2 \tag{4}$$

The Euler-Lagrange equations for the case under consideration are as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\varphi}} + \frac{\partial D}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = Q_\varphi^N \tag{5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{z}} + \frac{\partial D}{\partial \dot{z}} - \frac{\partial L}{\partial z} = Q_z^N \tag{6}$$

Subsequent derivatives obtained with the Euler-Lagrange equations are as follows:

$$\frac{\partial L}{\partial \varphi} = -2k_r l_r^2 \varphi \tag{7}$$

$$\frac{\partial L}{\partial z} = -2k_r z \tag{8}$$

$$\frac{\partial L}{\partial \dot{\varphi}} = I\dot{\varphi} \tag{9}$$

$$\frac{\partial L}{\partial \dot{z}} = m\dot{z} \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\varphi}} = I\ddot{\varphi} \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{z}} = m\ddot{z} \tag{12}$$

$$\frac{\partial D}{\partial \dot{\varphi}} = 2k_r l_r^2 \lambda \dot{\varphi} \tag{13}$$

$$\frac{\partial D}{\partial \dot{z}} = 2k_r \lambda \dot{z} \tag{14}$$

The results of the presented operations are used to determine the equations of motion of the system.

Equation of motion

Using the calculated derivatives, the equations of motions are based on the appropriate formulas. System equations of motions are described by the formulas (15)- (16)

$$F_{engine}l_r + I\ddot{\varphi} + F_{engine}l_r \cos\left(\Omega t\right) + 2k_r l_r^2 \varphi + 2k_r l_r^2 \lambda \dot{\varphi} = 0 \tag{15}$$

$$F_{engine} + m\ddot{z} - F_{engine} \cos\left(\Omega t\right) + 2k_r z + 2k_r \lambda \dot{z} = 0 \tag{16}$$

The determined equations constitute a mathematical dynamic description of the properties of the system. Further analysis allows for an effective analysis of the modeled object's operation and determination of its mechanical parameters.

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Determining fundemental matrix

The matrix of masses and stiffnesses of the system was determined from the equations of motion:

$$M = \begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \tag{17}$$

$$K = \begin{bmatrix} 2k_r l_r^2 & 0 \\ 0 & 2k_r \end{bmatrix} \tag{18}$$

The fundamental matrix, on the basis of which the characteristic equation of the considered system Δ was determined, is as follows:

$$A = \begin{bmatrix} -I\omega^2 + 2ik_r l_r^2 \lambda \omega + 2k_r l_r^2 & 0 \\ 0 & 2ik_r \lambda \omega + 2k_r - m\omega^2 \end{bmatrix} \tag{19}$$

$$\Delta = 4k_r^2 l_r^2 + Im\omega^4 - 2Ik_r\omega^2 - 4k_r^2 l_r^2 \lambda^2 \omega^2 - 2k_r l_r^2 m\omega^2 - 2iIk_r\lambda\omega^3 + 8ik_r^2 l_r^2 \lambda\omega - 2ik_r l_r^2 \lambda m\omega^3 \tag{20}$$

The fundamental matrix allows you to define a fixed solution. On the other hand, based on the characteristic equation, the eigenfrequencies of the system are determined.

General solution

General solution is presented by expression:

$$X_{g-\varphi(t)} = C_3 e^{-\frac{k_r l_r^2 \lambda t}{I}} \cos\left(t\sqrt{\frac{2k_r l_r^2}{I} - \frac{k_r^2 l_r^4 \lambda^2}{I^2}}\right) + C_4 e^{-\frac{k_r l_r^2 \lambda t}{I}} \sin\left(t\sqrt{\frac{2k_r l_r^2}{I} - \frac{k_r^2 l_r^4 \lambda^2}{I^2}}\right) \tag{21}$$

$$X_{g-z(t)} = C_1 e^{-\frac{k_r \lambda t}{m}} \cos\left(t\sqrt{-\frac{k_r^2 \lambda^2}{m^2} + \frac{2k_r}{m}}\right) + C_2 e^{-\frac{k_r \lambda t}{m}} \sin\left(t\sqrt{-\frac{k_r^2 \lambda^2}{m^2} + \frac{2k_r}{m}}\right) \tag{22}$$

General solution describes motion of the analised system - presents displacement i function of time - and is given by considerations about free vibrations of the system

Steady solution

The steady solution is given by the formula:

$$X_{s-\varphi(t)} = -\frac{0.5F_{engine}}{k_r l_r} - \frac{F_{engine} l_r \left(-\Omega^2 + \frac{2.0k_r l_r^2}{I}\right) \cos\left(\Omega t\right)}{I\left(4.0\left(-0.5\Omega^2 + \frac{k_r l_r^2}{I}\right)^2 + \frac{4.0\Omega^2 k_r^2 l_r^4 \lambda^2}{I^2}\right)}$$
$$- \frac{2.0F_{engine}\Omega k_r l_r^3 \lambda \sin\left(\Omega t\right)}{I^2\left(4.0\left(-0.5\Omega^2 + \frac{k_r l_r^2}{I}\right)^2 + \frac{4.0\Omega^2 k_r^2 l_r^4 \lambda^2}{I^2}\right)} \tag{23}$$

$$X_{s-z(t)} = \frac{0.5F_{engine}}{k_r} + \frac{F_{engine}\left(-\Omega^2 + \frac{2.0k_r}{m}\right)\cos\left(\Omega t\right)}{m\left(\frac{4.0\Omega^2 k_r^2 \lambda^2}{m^2} + 4.0\left(-0.5\Omega^2 + \frac{k_r}{m}\right)^2\right)} + \frac{2.0F_{engine}\Omega k_r \lambda \sin\left(\Omega t\right)}{m^2\left(\frac{4.0\Omega^2 k_r^2 \lambda^2}{m^2} + 4.0\left(-0.5\Omega^2 + \frac{k_r}{m}\right)^2\right)} \tag{24}$$

The specific solution is related to the presence of quantities that force motion (vibrations) of the analyzed system.